A Cellular Automaton Computer Model Supporting Quantum Field Theory

Hans H. Diel

Diel Software Beratung und Entwicklung, Seestr.102, 71067 Sindelfingen, Germany,
diel@netic.de

Abstract. This document describes a computer model of a cellular automaton (CA) that supports the modeling of quantum field theory (QFT). The computer model (and this document) supplements the authors work towards a functional model of QFT, including interactions in quantum theory. The computer model has the objective to support experimentation on alternative solutions and refinements of the functional model of QFT interactions.

1 Introduction

The computer model QFTCA is an implementation of a cellular automaton that supports the modeling of quantum field theory (QFT). The details of the particular area of QFT (e.g., quantum electrodynamics) are specified by the provision of a Lagrangian or other information related to the Lagrangian. The modeling of QFT requires, first of all, some "functional model" of QFT, i.e., a model that specifies the dynamical evolution of the system in terms of state transitions. Because such a functional model of QFT cannot be derived directly from the existing standard formulation of QFT, this functional model has to be formulated first. The functional model of QFT described in [1] and [2] is used as a basis for the computer model described in the present paper.

A major objective of the functional model of QFT and the described computer model is to include a model of measurement in quantum theory (QT). The QT measurement process, however, is not an explicit separate process within the model, but is realized by normal QFT/QT functions, primarily by QT/QFT interactions. Therefore, the modeling of QT/QFT interactions is the primary focused area in the described model.

Although the described cellular automaton model is mainly oriented towards QFT, other areas of physics are also supported, provided that they can be specified by a Lagrangian. Section 3 describes how some areas of classical physics are supported by the model. QFT Support is described in Section 4.

The computer model is an on-going project that has the goal to enable experimentation and refinement in areas that are difficult to analyze mathematically. Section 5 describes some of the planned areas of experimentation.

2 Overview

When Quantum Field Theory Cellular Automaton (QFTCA) is started, the window shown in Fig. 1 is displayed. For the further processing, three usage types can be distinguished:

1. Classical physics mode: continue input specification
2. QT/QFT example: press button "QT/QFT Example" (see Section 4)
3. Wave example: press button "Wave" (see Section 3.3)

2.1 Input Specifications

Fig. 1. Input Specification for CA.

The primary input is the Lagrangian or the equation of motion of the subject theory. After a Lagrangian or Hamiltonian [1] or the equation of motion is specified, the equation of motion (if not already specified) and the list of variables occurring in the equation of motion is generated by pressing the "Compile" button. Additional detailed specifications for the individual variables can be specified by pressing the "Initialize" button.

In addition, the following QFTCA visualization options can be varied by pressing the "V-Parms" button:

– title to be displayed with result graphics,

---

[1] at present not supported

- position of zero x-coordinate,
- position of zero y-coordinate,
- number of space dimensions to be visualized,
- coordinate of the space dimension to be displayed, if only a single space dimension is to be displayed.

2.2 Result of Running QFTCA

A QFTCA run shows the dynamical evolution of the system state. Visualization is provided for particle positions and the probability amplitudes associated with fields and particles.

2.3 Implementation

Starting with a Lagrangian, the equation of motion is determined by use of the Euler-Lagrange equation. This involves differentiation of the Lagrangian. The equation of motion is the basis for the update function of the CA. The equation of motion is typically a differential equation (often second order). In general mathematics, the computation of the state values $\psi(x,t)$ for fields and particles requires finding solutions for the respective differential equations. QFTCA includes neither a "general differential equation processor" nor a "symbolic mathematics engine". The transformation of the Lagrangian to the equation of motion supports only a limited variety of Lagrangians. The determination of the dynamical evolution of state variables such as ψ does not compute solutions of the differential equations but *interprets* the equation of motion in terms of the discrete state variables.

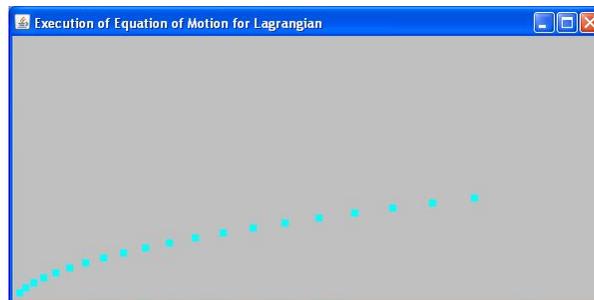# 3 Classical Physics

3.1 Classical (Newtonian) Mechaniscs

Fig. 2. Result of "Run CA" for "1/2*m* dv/dt = -F", F = - 1.

The input specification is shown in Fig. 1 for the Lagrangian L = T - V = 1/2 m v*v - F * x. After pressing the "Compile" button the equation of motion is determined to be "1/2*m* dv/dt = -F*1". From this equation of motion, a list of variables is extracted. For the example that results in Fig. 2, the variables are initialized as follows:

- tend = undefined
- x = 0
- v=dx/dt = (2,5)
- V(x) = F*x
- dV/dx = undefined
- m = 0.5
- F = -1

Invocation of "Run CA" (button) leads to a series of output visualization screens. A snapshot is shown in Fig. 2.
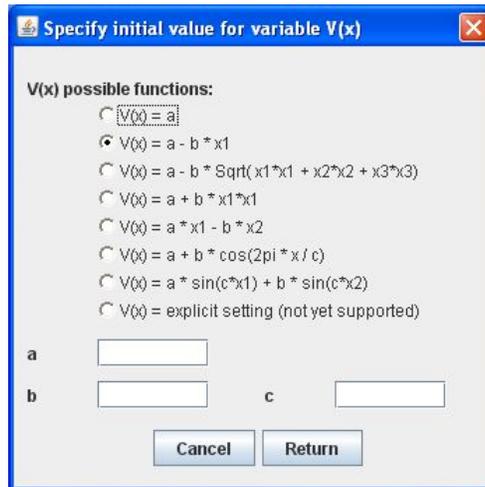

Fig. 3. Specification of initial values for field V(x).

Instead of specifying the potential field V directly as part of the Lagrangian, it is also possible to specify V(x) explicitly. An empty input for the entry field "Energy, position-related, V" (see Fig. 1) leads to an equation of motion "1/2*m* dv/dt = - dV/dx" and a corresponding variable list. Using the "Initialize" button, applied to V(x), the window shown in Fig. 3 is invoked and offers a number of field initialization functions. For example, if "V(x) = a - b*x1" is selected with a=3, b=1, then the field is initialized accordingly. "Run CA" displays the curve shown in Fig. 4 as a function of the specified Lagrangian, initial variable values and initial field specification.

It is also possible to specify the equation of motion directly instead of having QFTCA derive it from the Lagrangian.
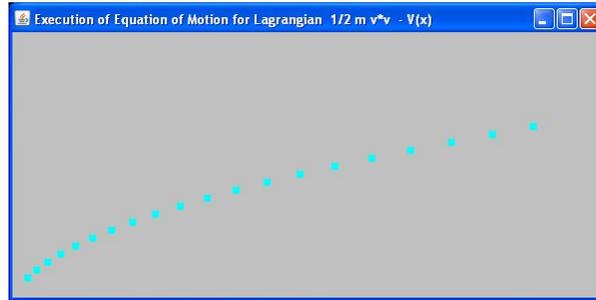
Fig. 4. Result of "Run CA" for "1/2*m* dv/dt = V(x)", V(x) = 3 - x.
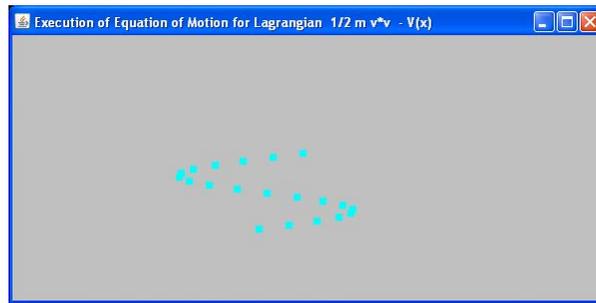
3.2 Harmonic Oscillator

Fig. 5. CA run for harmonic oscillator $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$.

A particle of mass m undergoing simple harmonic motion is described by the Lagrangian

$L = T - V = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ (see, for example, [7], page 25).

As with Newtonian mechanics (Section 3.1), the CA for the harmonic oscillator can be obtained by the specification of this Lagrangian or by entering the equation of motion directly as " $d^2x/dt^2 = -k/m \cdot x$". The oscillation shown in Fig. 5 was achieved with the following initial variables

- t0 = 0
- tend = undefined
- x = 0
- v=dx/dt = (60,4)
- V(x) = see below
- dV/dx = undefined
- k = (0.033, 0)
- VParms: zeroXcoord = 250, duration = 50

The V-part of the Lagrangian can be specified directly (i.e., $V = \frac{1}{2}kx^2$) or can be specified by a suitable explicit field V(x). A field that can cause a harmonic oscillation has to contain a symmetric depression, for example "V(x) =300 - 0.033 x*x". Fig. 6 shows a CA run with such a V(x) field.

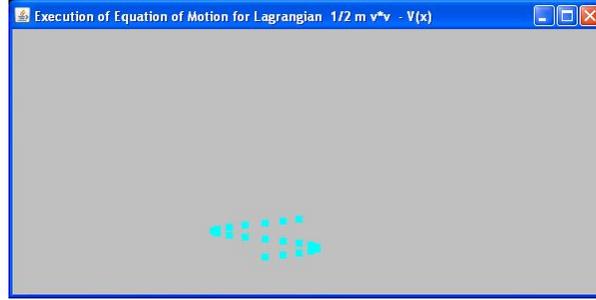**Fig. 6.** Result of "Run CA" for harmonic oscillator with "V(x) =300 - 0.033 x*x" .

3.3 Waves

For waves, we start immediately with the wave equation (i.e., the equation of motion) instead of the Lagrangian, which is

(1) $(\frac{1}{v^2}\frac{d^2}{dt^2} - \frac{d^2}{dx^2})\psi(x,t) = 0.$

The three-dimensional version is

(2) $(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \bigtriangledown^2)\psi(x,t) = 0.$

Depending on the particular context, the equation may be varied or extended by setting the right hand side unequal to 0. For example, in [8] the equation of motion for class 1 waves is

(3) $d^2\psi/dt^2 - c_w^2 d^2\psi/dx^2 = (2\pi\nu)^2(\psi - \psi_0).$

QFTCA supports the modeling and visualization of waves in one, two, or three space dimensions through the parameter "Number of space dimensions" shown in Fig. 1. [2]

Fig.7 shows a snapshot from the dynamical evolution of a wave in one space dimension according to the equation of motion and a suitable initial state.
Fig.8 shows the same wave in two space dimensions.

The impressive power and compactness of the wave equation (equation (1) or (2)) means that the wave equation applies to a large variety of wave types, wave forms, and wave dynamical evolutions. However, this also means that the differentiation has to come from parameters outside the wave equation. For the CA-based functional model QFTCA this external influence comes primarily from the initial value settings for the wave. QFTCA supports the specification of the initial wave in the window shown in Fig. 3. The wave displayed in Fig. 7 and Fig.

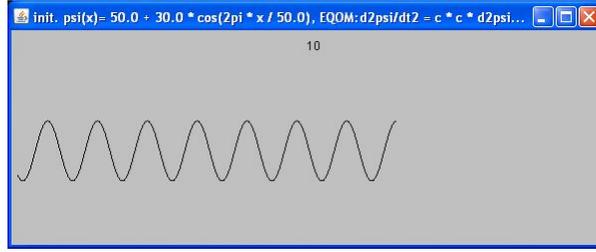[2] Support of three space dimensions is not yet implemented.

Fig. 7. 1-dimensional visualization of wave $(\frac{1}{v^2}\frac{d^2}{dt^2} - \frac{d^2}{dx^2})\psi = 0$
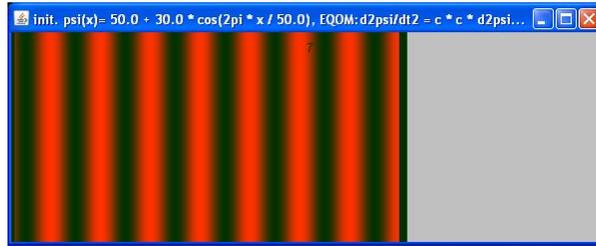

Fig. 8. 2-dimensional visualization of wave $(\frac{1}{v^2}\frac{d^2}{dt^2} - \frac{d^2}{dx^2})\psi = 0$

8 has been initialized using the option "V(x) = a + b * cos(2pi * x / c)" with a=50, b=30, c=50. In contrast to the field initializations described in Sections 3.1 and 3.2, the field initialization for waves (and for fields described in Section 3.4), has to not only set the initial values of $\psi(x, t_0)$, but also has to provide initial values of $d\psi/dx$.

3.4 Fields

Two types of fields are addressed in QFTCA

1. The static fields V(x) appearing in the Lagrangians of Newtonian mechanics (Section 3.1) and of the harmonic oscillator (Section 3.2).
2. The dynamic fields $\psi(x, t)$ based on waves.

This section focuses on dynamic fields $\psi(x, t)$. Support of static fields V(x) has been addressed in Sections 3.1 and 3.2.

The Lagrangian or the equation of motion for dynamic fields contains derivatives $d\psi/dx$ and/or $\frac{d^2\psi}{dx^2}$. Support and visualization of the dynamical evolution of a field by QFTCA requires storing the first derivative $d\psi/dx$ for all (x, t), in addition to the field values $\psi(x, t_0)$.

Fig. 8 and Fig. 9 show two examples of two-dimensional field visualization of static fields V(x) or initial dynamic fields. Fig. 8 results from the initialization option "V(x) = a + b * cos(2pi*x/c)" shown in Fig. 3. Fig. 9 results from "V(x) = a * sin(c*x1) + b * sin(c*x2)" shown in Fig. 3.
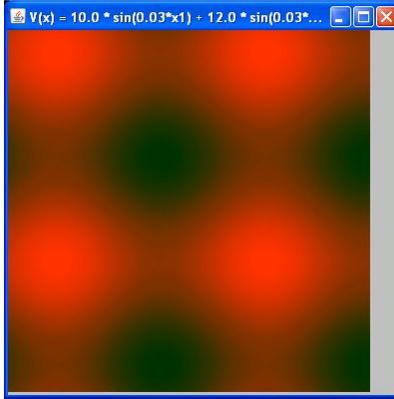
Fig. 9. 2-dimensional visualization of field "V(x) = a * sin(c*x1) + b * sin(c*x2)" .

4 Quantum Field Theory

The QFT session is invoked by the "QT/QFT Example" button at the window shown in Fig. 1. This leads to the prompting for the relevant input parameters. Fig. 10 shows the QFTCA window for the specification of the input parameters.

The specification starts with the selection of the QFT area. Currently, QFTCA only supports interactions in quantum electrodynamics (QED) and weak interactions. After the QFT area has been selected, the related Lagrangian and equation of motion can be generated by pressing the button "Generate Lagrangian". In the final QFTCA it will also be possible for the user to directly specify the Lagrangian. In that case the button "Lagrangian → EQM" can be used to generate the equation of motion from the Lagrangian. [3] Once the QFT area, the Lagrangian and the equation of motion are known, the interacting particles and fields have to be specified. QFTCA supports interactions between two particles or between a particle and a field. The user specifies the interacting particles/field by entering a name in the field "Particle 1" or "Particle 2" or "Fieldname" and pressing the related "Details of ..." button. This leads to a separate window for the specification of the particle/field details such as, for example, the particle type. The specification of particle attributes can be eased by specific global specifications. "CoM frame" means that "center of mass" is assumed for the momentum specifications. "Momentum in z-direction" sets the momenta in the other directions (i.e. x and y) to 0.

After the interacting particles/field have been specified, the "List" button causes the possible interaction results to be displayed. For example, QFT predicts that electron-positron scattering (i.e., Bhabha scattering) may result in an electron and a positron, a muon-antimuon pair, or a tauon-antitauon pair. In

---

[3] To enter and display the mathematical equation for the Lagrangian and the equation of motion a special notation that is limited by the available character set is used. This notation is not explained here because it is not important for the present document.

Fig. 10. QFTCA - Parameter Specification for "Start QFT CA" mode.

real nature, the determination of the "out" particles is a non-deterministic process, and QFTCA could simulate this non-determinism by the invocation of a "random()" function. Assuming that the QFTCA user is interested in a specific case, QFTCA prompts the user for the selection of a specific "out" alternative before the simulation of the QFT interaction can be invoked for the specified parameters.

The further execution of the QFTCA computer model can be invoked by pressing the "Run CA" button, "Show Energy dep.", or "Show Angle dep." button. "Run CA" shows the stepwise CA progression starting with the specified initial CA state, including the interaction, and ending with some "out" state. "Show Energy dep." and "Show Angle dep." do not show any intermediate states but only a final state in terms of probabilities (or probability amplitudes) or differential cross-section. Depending on the selected particular button, the result probabilities(or differential cross-sections) can be obtained either for a range of energy values or angle values. For example, requesting "Show Angle dep." leads to the visualization of the differential cross-section as a function of the scattering angle as shown in Fig. 11.

Before the visualization of the results of a QFTCA run starts, the user is prompted the specification of the spin values of the outgoing particles. QFTCA supports visualization for a specified spin value combination or for the sum of all possible spin combinations.

An example for the visualization of energy dependency is shown in Fig. 13. The figure shows the results of Bhabha scattering (as with the previous examples) for the scattering angle = 50 and an energy range 5 to 30 MEV.

As can be seen in the "List of possible out alternatives" in Fig.10, electron - positron scattering may also result in out particle combinations other than electron, positron. Selecting tauon-pair creation ($P1 + P2 \rightarrow Tauon-, Tauon+$) and sufficient total energy, such as 10 GeV results in the angle dependent probabilities is shown in Fig. 16.

Section 4.2 shows some major examples of QFT scatterings.

4.1 Program Logic

The purpose of this section is not to describe any implementation techniques but to show the mapping of the QFT computation rules to the QFTCA program logic.

Generation of the list of possible "out" alternatives After the input parameters have been specified, pressing the "List" button causes the generation of a list of possible out alternatives. As described in [1], the QFTCA functional model assumes the generation and processing of "interaction channels" to be a major process step within a QFT interaction. An interaction channel starts with the two "in" particles (e.g., electron P1 and positron P2 in the example in Fig. 10) and proceeds with a pair of split() and combine() operations. For the example, as shown in Fig. 10, the following interaction channels, i.e., pairs of split() and combine() operations, are possible.
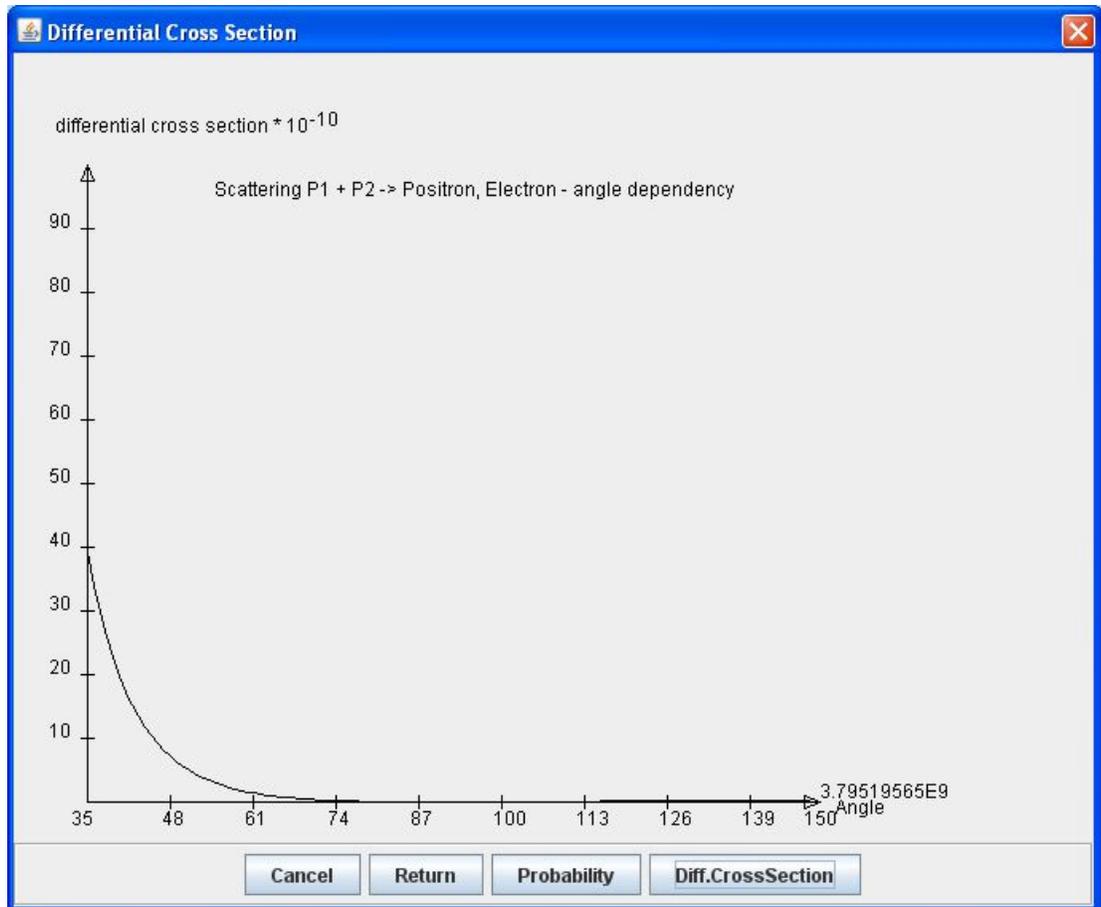
Fig. 11. Differential cross-section for electron - positron scattering as a function of scattering angle.

1. $split(P1) \to (e^-, \gamma); combine(P2, \gamma) \to e^+$
2. $split(P2) \to (e^+, \gamma); combine(P1, \gamma) \to e^-$
3. $split(P2) \to (e^+, \gamma); combine(P1, e^+) \to \gamma$
4. $combine(P1, P2) \to \gamma; split(\gamma) \to (e^-, e^+)$
5. $combine(P1, P2) \to \gamma; split(\gamma) \to (\mu^-, \mu^+)$
6. $combine(P1, P2) \to \gamma; split(\gamma) \to (\tau^-, \tau^+)$

Each of these six possible interaction channels corresponds to a possible Feynman diagram and the set of these interaction channels has to correspond to the set of possible Feynman diagrams for the "in" particles P1 and P2. For example, case (1) $split(P1) \to (e^-, \gamma); combine(P2, \gamma) \to e^+$ and case (4) $combine(P1, P2) \to \gamma; split(\gamma) \to (e^-, e^+)$ correspond to the Feynman diagrams shown in Fig. 12.

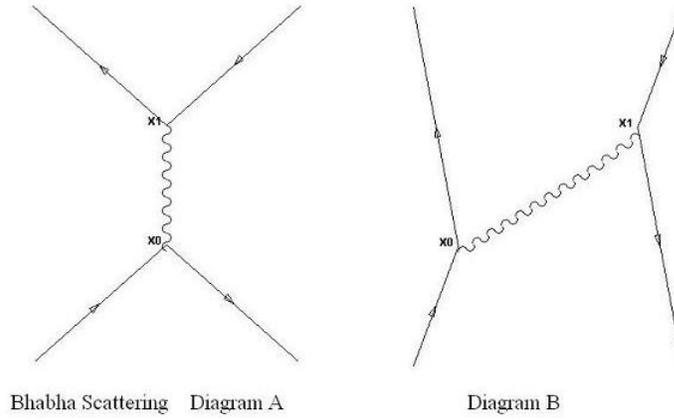Bhabha Scattering   Diagram A   Diagram B

Fig. 12. Feynman diagrams corresponding to interaction channels (1) and (4).

Based on the above list of possible interaction channels, the QFT interaction between P1 and P2 may result in the following four alternative "out" particle combinations: $((e^-, e^+), (\gamma, \gamma), (\mu^-, \mu^+)$ and (τ^-, τ^+).

Identification of fermion chains The QFT equations for the evaluation of the probability amplitudes for the scattering matrices are based on fermion chains. Therefore, after the interaction channels have been identified, the fermion chains pertaining to the interaction channels must be identified by QFTCA.

Elimination of equivalent interaction channels With the above-described example, the three interaction channels are:

(1) $split(P1) \to (e^-, \gamma); combine(P2, \gamma) \to e^+,$
(3) $split(P2) \to (e^+, \gamma); combine(P1, e^+) \to \gamma$ and

(4) $combine(P1, P2) \rightarrow \gamma; split(\gamma) \rightarrow (e^-, e^+)$.

all result in the same "out" particle combination (e^-, e^+). According to the QFT rules, cases (1) and (3) are equivalent, and therefore one of them (e.g. case (3)) must be dropped. Cases (1) and (4) must be continued, and their final amplitudes must be added. In QFTCA, the determination of equivalent (i.e. redundant) interaction channels is primarily performed by a comparison of the respective fermion chains.

Processing of interaction channels The interaction channels pertaining to the selected output configuration are processed in parallel and the resulting probability amplitudes are added. For example, if in Fig.10 the alternative "P1 + P2 $\rightarrow$ Positron, Electron" (i.e. Bhabha scattering) is selected, the two interaction channels "$split(P1) \rightarrow (e^-, \gamma); combine(P2, \gamma) \rightarrow e^+$" and "$combine(P1, P2) \rightarrow \gamma; split(\gamma) \rightarrow (e^-, e^+)$" have to be processed. The processing starts with the unique "in" state specified in the window shown in Fig. 10. The "out" state is varied with respect to energy/momentum, scattering angle, and spin.

4.2   Scattering Examples

Bhabha scattering Fig. 11 shows the differential cross-section as a function of the scattering angle for Bhabha scattering (i.e. electron-positron scattering). The results shown in Fig. 11 have been obtained with the following parameter settings at the window shown in Fig. 10:

– Energy (range): 5 MeV
– Angle between particles: 35 - 150
– CoM frame: yes
– Particle1, particle2: Electron, Positron
– Spins: all spins = 1 (not shown in Fig. 10)
– Selected out alternative: "P1 + P2 $\rightarrow$ Positron, Electron"
– Chosen result option: Differential cross-section

Fig. 13 shows the probability distribution as a function of the energy range 5 - 30 MeV.

Moeller scattering Fig. 14 shows the probabilities as a function of the scattering angle for Moeller scattering (i.e. electron-electron scattering). The results shown in Fig. 14 have been obtained with the following parameter settings at the window shown in Fig. 10:

– Energy (range): 500 MeV
– Angle between particles: 5 - 175
– CoM frame: yes
– Particle1, particle2: Electron, Electron
– Spins: all spins = 1 (not shown in Fig. 10)
– Selected out alternative: "P1 + P2 $\rightarrow$ Electron, Electron"
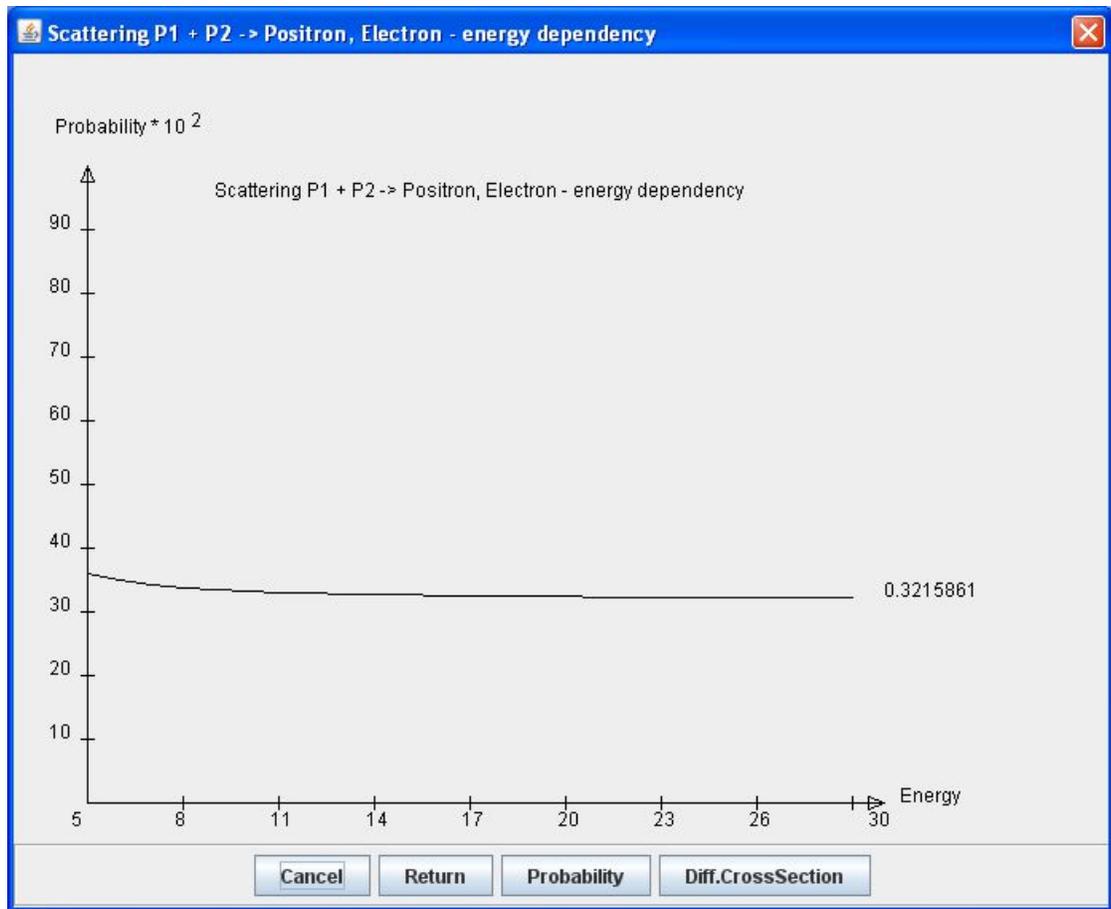– Chosen result option: Probability

Fig. 13. Probabilities for electron - positron scattering as a function of energy.
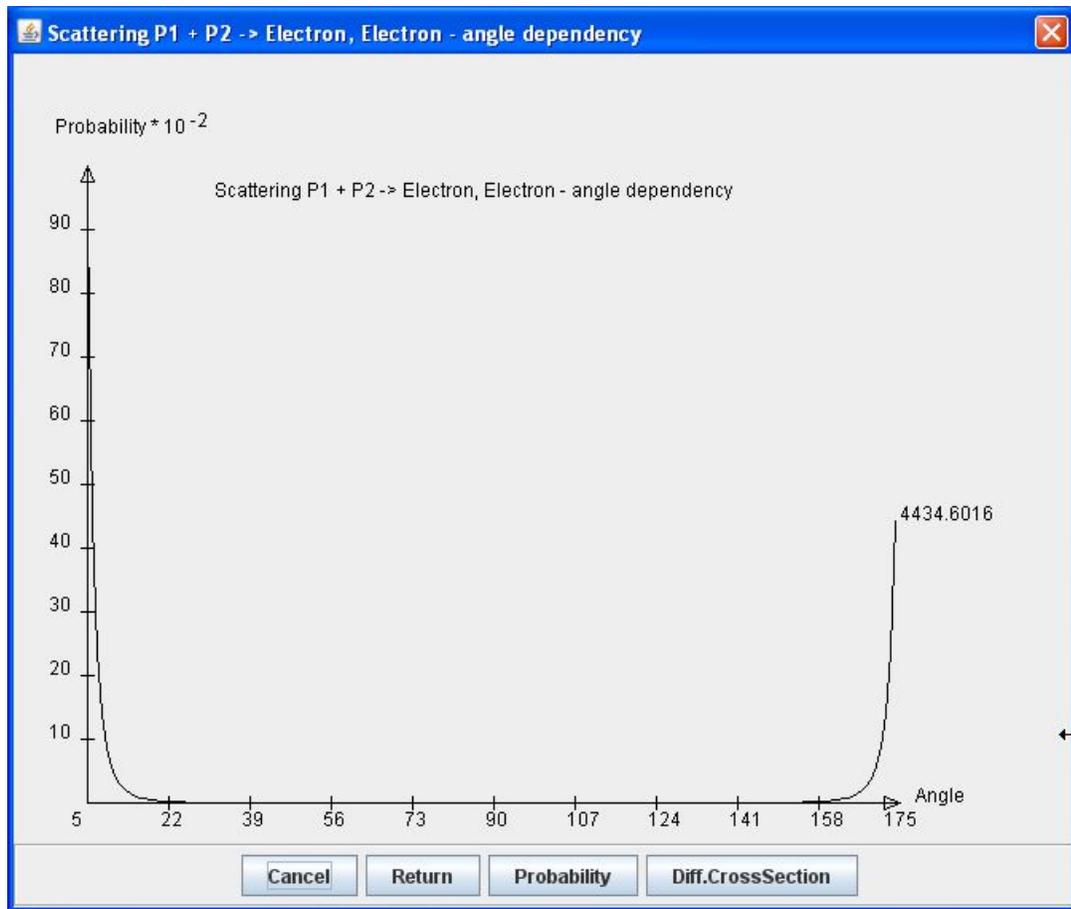
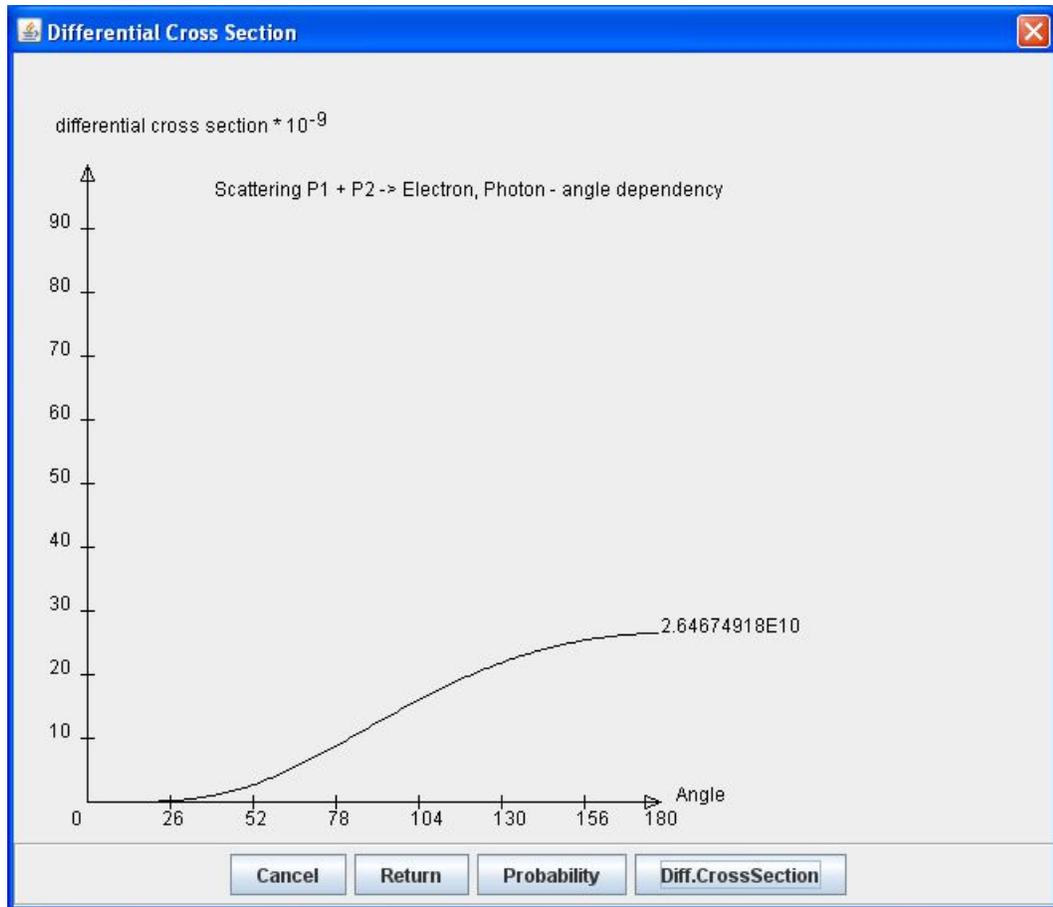**Fig. 14.** Probabilities for electron - electron scattering as a function of scattering angle.

Fig. 15. Differential cross-section for photon - electron scattering as a function of scattering angle.

Compton scattering Fig. 15 shows the differential cross-section as a function of the scattering angle for Compton scattering (i.e. photon-electron scattering). The results shown in Fig. 15 have been obtained with the following parameter settings at the window shown in Fig. 10:

– Energy (range): 0.511 MeV
– Angle between particles: 0-180
– CoM frame: NO
– Particle1, particle2: Photon, Electron
– Spins: all spins = 1 (not shown in Fig. 10)
– Selected out alternative: "P1 + P2 → Photon, Electron"
– Chosen result option: Differential cross-section

Tauon pair creation Fig. 16 shows the probabilities as a function of the scattering angle for tauon pair creation. Tauon pair creation is another possible result of electron-positron scattering, if sufficient energy is available.The results shown in Fig. 16 have been obtained with the following paramter settings at the window shown in Fig. 10:

– Energy (range): 10000 MeV
– Angle between particles: 0 - 180
– CoM frame: yes
– Particle1, particle2: Electron, Positron
– Spins: all spins = 1 (not shown in Fig. 10)
– Selected out alternative: "P1 + P2 → Tauon-, Tauon+"
– Chosen result option: Probability

5 Planned Experimentations

Major objective of the QFTCA computer model is to support experimentation with questions that can hardly be answered by purely theoretical thinking. The major subjects for such experimentations are the following:

5.1 General aspects

Granularity of discreteness It will be interesting to study how different granularities of the discreteness of state parameters affect the simulation results. Special attention is devoted to the granularity of the path set resulting from a QFT interaction.

Transition of probabilities to facts In contrast to standard QT/QFT, the functional model QFTCA defines explicit points during the interaction process where the probability amplitudes are turned into facts. The introduction of the respective process steps has been designed with the goal that no or only minimal deviations from standard QT/QFT should be detectable. The executable QFTCA may be used to verify to what extend this goal has been accomplished.
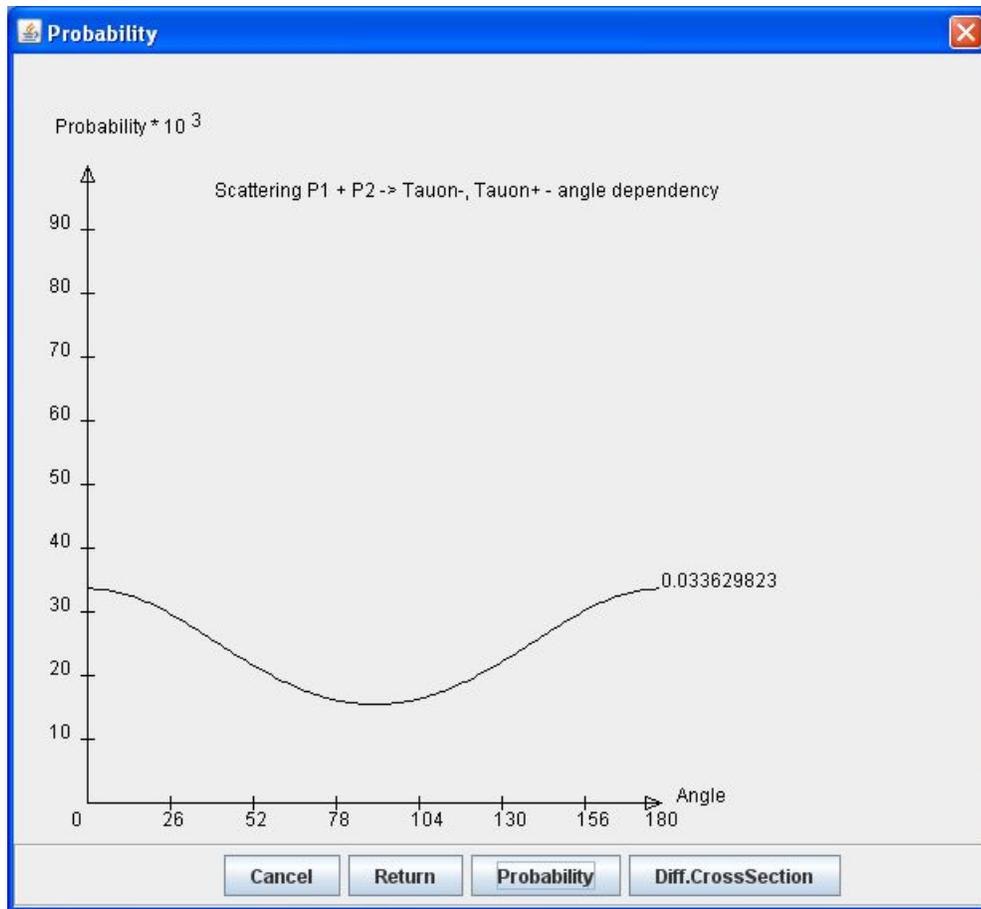
18

Fig. 16. Probabilities for $P1 + P2 \rightarrow Tauon-, Tauon+$.

Position space versus momentum space In QFT, two types of computations of scattering amplitudes are distinguished: computation in momentum space and computation in position space. Computation in momentum space is the dominating type of computation in QFT textbooks and is also used in the computer model QFTCA. For an increased exploitation of the cellular automaton concept (partial) support of computation in position space is intended.

Extended utilization of waves The major goal of the QFTCA computer model is to support QFT interaction. For an increased CA exploitation with QFT interactions (as described in Section 5.2), additional features of waves have to be studied. The major subjects will be: more complex wave equations, moving waves, superpositions, and interacting waves.

5.2 Increased CA exploitation with interactions

The provision of a functional model of QFT interactions, which is based on a cellular automaton, resulted to several benefits. However, in order to maintain compatibility with standard QFT, the degree of CA exploitation is, so far, relatively weak. For example, the complete QFT interaction is mapped to a single CA update cycle. An increased CA exploitation is a major objective with further QFTCA development.

5.3 Functional model for the reduction of path-set of a particle/wave collection

A very important part of the model of measurement process described in [5] is the reduction (i.e. collapse) of the path set. This reduction of the path set may be considered as a non-local operation. Non-local models are not appreciated by physicists. The non-local model also disturbs the overall nature of a cellular automaton. Possible solutions for a reduction or elimination of the non-locality will be analyzed.

5.4 Reducing the non-locality of the entanglement model

The functional model of interactions described in [2] includes a model of QT entanglement based on "particle/wave collections". The "particle/wave collections" may be considered as non-local objects. Possible solutions for a reduction or elimination of the non-locality will be analyzed.

6 Discussion

6.1 Accuracy of the Results

Common question with all computer models is the accuracy of the modeling results. For QFTCA, this question has been reduced to the question to what

extent the model results are in accordance with the results predicted by the physics theory, especially the results predicted by QFT. During the development of QFTCA (and of the predecessor model described in [6]), the results generated by QFTCA have been compared with comparable predictions published in literature or in the Internet. Because of the lack of comparable data, these tests of QFTCA, however, have not been very extensive. The authors main interest was to ensure that the overall concept works and produces (wherever comparable data was available) results that are compatible with standard QFT. This may include some "untested" areas in the present computer model.

7   Summary and Conclusions

This document describes a computer model that supports the modeling of quantum field theory. The computer model QFTCA represents a "functional model" of QFT, i.e. a model that describes QFT processes in terms of state transitions. The major objective of QFTCA is to demonstrate that such a functional model of QT/QFT is feasible at all. At the present stage of completion, it can be said that a functional model has been achieved. Yet, additional, more detailed modeling of state transitions at lower levels would be desirable and are under development.

In the course of model development, the author encountered a number of problems inherent to the translation of quantum theory into a computer program. The problems encountered are described in [3] and, more generally, in [4]. Major observations and conclusions are:

1. A satisfactory solution to the measurement problem of QT and a satisfactory "interpretation of QT" can be probably achieved by a functional interpretation of QT (see [5]).
2. The lack of functional descriptions is a problem with not only quantum theory but also with other areas of physics that address non-trivial processes.
3. Quantum physics is not computable in several areas. The non-computability of QT, however, is neither a consequence of the indeterminism within QT nor a consequence of the strangeness of QT. The non-computability is caused by some improper formulations within the theory.

References

1. H. Diel: "A Lagrangian-driven Cellular Automaton supporting Quantum Field Theory", http://arxiv.org/abs/1507.08277 (2015)
2. H. Diel: "A functional model of interactions in quantum theory", arXiv 1405: 6099v2 (2015)
3. H. Diel: "Problems and Possible Solutions with the Development of a Computer Model of Quantum Theory", World Academy of Science, Engineering and Technology 74 (2013)
4. H. Diel: "On the Computability of Quantum Theory", International Journal of Computational Physical Sciences. ISSN 0976-5875 Volume 4, Number 1 (2013), pp. 7-22

5. H. Diel: "A model of the measurement process in quantum theory", arXiv1405:5982v1 (2014)
6. H. Diel: A Computer Model of Quantum Field Theory, ICCQMNP Berlin, 2013
7. D. McMahon: Quantum Field Theory Demystified
8. M. Strassler: Conversations About Science with Theoretical Physicist M. Strassler, http://profmattstrassler.com/articles-and-posts/particle-physics-basics/Higgs/6. Fields Of Particular Significance.html

A   Completions

1. QFT "Run CA"
2. Decays
3. JPG - jpg ?

B   Open questions

1. Determination of Evalfactor; When are diagrams equivalent ? Explanation given in Mac Mahon, page 178 cannot be correct ?
2. Determination of Evalfactor; +/- rule ?
3. Spinor computation for fractional spin values ?
4. Why is Compton scattering with positron exchange not part of computation?
5. Spin conservation?
6. QFT interaction?